21.

$$\int_{-\infty}^{\infty} e^{-|x|}\,dx = \int_{-\infty}^{0} e^{-|x|}\,dx + \int_{0}^{\infty} e^{-|x|}\,dx$$

$$= \lim_{b\to-\infty}\int_{b}^{0} e^{-(-x)}\,dx + \lim_{b\to\infty}\int_{0}^{b} e^{-x}\,d \qquad |x| = \begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$$

$$\lim_{b\to-\infty} e^{x}\Big|_{b}^{0} + \lim_{b\to\infty} -e^{-x}\Big|_{0}^{b}$$

$$\lim_{b\to-\infty}\left(e^{0}-e^{b}\right) + \lim_{b\to\infty}\left(-e^{-b} - -e^{-0}\right) = 2$$

$$1-0 \quad + \quad 0+1$$

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23.

$$\int_{-\infty}^{\infty}\frac{dx}{e^{x}+e^{-x}} = \int_{-\infty}^{0}\frac{dx}{e^{x}+e^{-x}} + \int_{0}^{\infty}\frac{dx}{e^{x}+e^{-x}}$$

$$\lim_{b\to-\infty}\int_{b}^{0}\frac{1}{e^{x}+e^{-x}}\,dx + \lim_{b\to\infty}\int_{0}^{b}\frac{1}{e^{x}+e^{-x}}\,dx$$

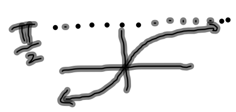
$$\int\frac{1}{\left(e^{x}+\frac{1}{e^{x}}\right)}\frac{e^{x}}{e^{x}}\,dx = \int\frac{e^{x}}{(e^{x})^{2}+1}\,dx = \int\frac{du}{u^{2}+1} = \tan^{-1}u$$

$$= \tan^{-1}(e^{x})$$

let $u = e^{x}$

$du = e^{x}\,dx$

$$\lim_{b\to-\infty}\tan^{-1}(e^{x})\Big|_{b}^{0} + \lim_{b\to\infty}\tan^{-1}(e^{x})\Big|_{0}^{b}$$



$$\lim_{b\to-\infty}\left(\tan^{-1}e^{0} - \tan^{-1}e^{b}\right) + \lim_{b\to\infty}\left(\tan^{-1}(e^{b}) - \tan^{-1}e^{0}\right)$$

$$\frac{\pi}{4} - 0 \quad + \quad \frac{\pi}{2} - \frac{\pi}{4} \quad = \frac{\pi}{2}$$

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8.4 b methods of comparison

if $\int_a^\infty f(x)\,dx \le \int_a^\infty g(x)\,dx$

converges

then $\int_a^\infty f(x)\,dx$ converges

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if $\int_a^\infty f(x)\,dx \le \int_a^\infty g(x)\,dx$

diverges

then $\int_a^\infty g(x)\,dx$ diverges

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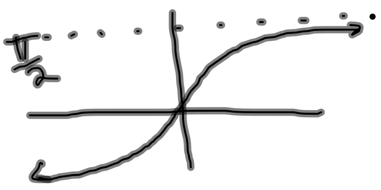
$$0 \le \int_1^\infty \frac{dx}{x^3+1} \le \int_1^\infty \frac{1}{x^2+1}\,dx$$

must converge by comparison

Converges to $\frac{\pi}{4}$

$$\lim_{b\to\infty} \int_1^b \frac{1}{x^2+1}\,dx$$



$$\lim_{b\to\infty} \tan^{-1} x \Big|_1^b$$

$$\lim_{b\to\infty} \left(\tan^{-1} b - \tan^{-1} 1\right)$$

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

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$$\int_1^\infty \frac{1}{\sqrt{x^4+1}}\,dx \leq \int_1^\infty \frac{1}{\sqrt{x^4}}\,dx$$

prove this converges

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